Valuation Report Preparation Guide

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Note to PVX users: This guide is intended to aid you in writing fractional interest valuation reports. For this guide to be of use, you need to have a subscription to PVX and either a hard copy or digital copy of *Valuing Fractional Interests in Real Estate 2.0*, by Dennis A. Webb. (If you are already a PVX subscriber, you have a digital copy of the book at your fingertips.) This guide will frequently refer you to one source or the other. If a reference is a numbered question, it's pointing you toward the PVX Dashboard. References to case illustrations and pages are pointing you toward the book. At the end of this document is a simplified outline containing all of the references you will see in the following pages.

Building your report is a key part of your overall learning process. As you interact with PVX, you will become familiar with the book's 2.0 valuation technology at a level that will support solid and convincing evidence for value. Of course, it will take some work, but the more familiar you are with the concepts, the faster it will go.

Your pre-qualifications for this task are that you are a professionally designated and experienced real estate appraiser or business valuer, and that you have a working familiarity with the income approach. PVX does not try to replicate the bodies of knowledge that have been developed for the established valuation disciplines, including the income approach. PVX does present the 2.0 principles and processes in their entirety, as they essentially constitute a third professional discipline. The connections between 2.0, business valuation and real property appraisal are detailed throughout the book, along with the processes of 1.0. You are encouraged to begin by reading through the first few chapters of the book and the descriptions of 1.0 alternatives found in chapter 6 (Public Limited Partnership Secondary Market), chapter 7 (Methods for Developing the Discount), and chapter 10 (Methods for Developing the Discount). You may even wish to incorporate some of these original processes in your work as reasonableness checks for PVX results.

The purpose of the valuation report is to tell your story of value. This outline is comprehensive, including much more explanation and detail than is absolutely required. For example, you could simply footnote book sections rather than extracting them and adding to your report, thus shortening it quite a bit. (Some of the longer explanations can be relegated to the appendix, as recommended throughout this document.) However, you are obligated to provide enough information such that a valuer with sufficient qualification could reproduce your analysis and conclusions. So don't shortcut too much. The good news is that you will learn a great deal by going through this process carefully, and most of what you prepare will change little, if at all, from report to report. So, it is mostly one and done.

Suggested text for inclusion in your own report is provided directly for many sections in *italic font with grey fill*. You are otherwise directed to the book's case study illustrations and a few complete documents that you can adapt for your report.

If mastery is your goal (and it's not very far away), this site has everything you need to get there, including several decades of this author's experience condensed for your benefit. You are well on your way to mastering this multidisciplinary type of valuation. Have a good journey.

Source Data and Guidance

Your interface with PVX is the Dashboard, which has been designed to facilitate fact discovery and capture your inputs, observations and comments as you go. So far, so good, but not yet a report. The user valuation summary (UVS) then outputs the entirety of the Dashboard: numerical inputs (or defaults), your comments and notes, internal formulas, calculations, discounted cash flow tables, and other information from the algorithm's valuation models. Your report will use the UVS as source material, but your report has a different job: communicating the results of your valuation to the report user.

The book (from Part II onward) is your guide to report construction. It is based on two case studies, complete with numerous examples of how facts, analysis and results can be communicated in a report. The book is meant to show general application of both 1.0 and 2.0 processes, with a greater focus on 2.0. Although the successive case studies essentially constitute a complete valuation report, the book does not follow PVX organization exactly. The outline below follows a recommended report structure, identifying book sources of descriptive text (generally considered boilerplate, although some will have to be modified for different entity types and some property types) and samples of case-specific text, referencing book pages so you can extract what you need. Please read carefully, and study further when you hit unfamiliar material. It is all here, but you might want to acquire additional references that you don't already have. I recommend using references in the book as citations for the same material extracted for your report.

Authority

Your report will have many elements that rely on established procedures, methods and best practices, as it must to conform with the *Case of Daubert* standard. It must also be sufficiently complete that another qualified valuer could follow your analysis and arrive at the same conclusion. Thus, footnotes are essential. Fortunately, *Valuing Fractional Interests in Real Estate 2.0* contains the entire relevant body of knowledge for this practice, including traditional 1.0 methods that you might also be using. You can cite it directly, or any of the 17 books, 42 articles, seven online resources and seven court cases in its bibliography.

SPECIAL NOTE: Your authority is NOT PrimusPVX, any more than you would cite Microsoft Excel as an authority. Both are tools. PVX was developed in Excel, and an Excel model is used for software coding quality control. PVX does contain databases, but, for example, the REIT Yield Premium Database is described on pages 148–161 of the book and was further described in the peer-reviewed article: Dennis A. Webb, "Using the Income Approach for Minority Interests," *The Appraisal Journal*, (Chicago: Spring 2018): 125–136. Either of these sources would be your appropriate citation.

Likewise, the labels "2.0" and "1.0" are used in the book and in PVX to distinguish traditional business valuation methods from the income approach, but they are not themselves terms of art or otherwise references to authority. You should refer to the underlying methods directly, without classifying them as belonging to 2.0 or 1.0.

Front Matter

The title page, letter of transmittal, executive summary (if you wish to include one) and limiting conditions should follow your desired report format. (These are not modeled in the book's case studies.) The subject of the report is the interest (or interests) being valued, which should be defined in your letter of engagement. The **letter of transmittal** should mirror the engagement, describing the subject interest, entity, real estate, purpose and use, and any extraordinary conditions or circumstances that you want to emphasize, such as hypothetical conditions, along with the conclusion of value.

The **executive summary** is meant to highlight basic analysis and conclusions so the reader can understand the scope of the valuation, but with limited explanation. It is not absolutely necessary, but is usually very helpful for the reader, since the report is likely going to be pretty long. (There is no prize for length per se, so if you prefer a shorter report, the summary may not be necessary.)

Limiting conditions are generally for business valuation (conforming with USPAP Standard Rule 10, for example), but will also include conditions applicable to the real estate if you are combining the real estate appraisal with your report. Otherwise, real estate limiting conditions (conforming with Standard Rule 2) belong in the real estate appraisal document. A business valuer would not include real estate limiting conditions in their valuation report.

I. Introduction

Introductory detail is shown in **Case 1 Illustration 3.1**, **p. 66**.

Subject Ownership Interest

Repeat the subject interest description (type and percentage interest being valued, its entity's legal description, etc.) and identify the real estate, including defined terms for each, such as "subject interest," "Company" (for LLCs) or "Partnership" (for LPs), "Property" for the real estate, or more specific terms, as you wish.

Purpose & Use

Repeat the purpose and use from the engagement, along with the intended users, the compliance statement, and your competency statement, if needed.

The purpose of the introduction is to identify the interest being valued with a little context, along with the purpose and use of the valuation.

Date(s) of Value

The effective date of value, along with the date of property inspection and of the report, if you wish.

Definitions

For the standards of value, you can adopt text from **pp. 44–49** or use others as appropriate. See the relevant section of **Case 1 Illustration 3.1, p. 68**. For the definitions of the subject interests, you might want to use text from **pp. 54**–**55**, including any other needed definitions, such as: minority interest, assignee interests, and any real estate definitions that are material (e.g., fee simple, leased fee). See the relevant section of **Case 1 Illustration 3.1, p. 67**. Real estate definitions will, of course, be included in any separate report, but the reader's understanding of any important distinctions can be enhanced if they are repeated here.

Valuation Approaches/Scope of Work

As usual, the scope of work, on **pp. 55–59**, should summarize your information and data sources (although you can forego sources that are footnoted elsewhere in the report), and identify the methods you used. Methods are only summarized here, as they will be discussed in detail in the valuation analysis section (III) of the report. Be sure to break down methodology by "level of value," beginning with the real estate appraisal, entity (balance sheet) valuation, minority-marketable (or just minority) level and nonmarketable level. Include any needed disclaimers concerning limitations on information availability or reliability and client attestations.

II. Entity, Property & Economic Conditions

Suggested report text:

> *This section concerns the legal, circumstantial and external economic/financial facts and conditions that underlie our understanding of the rights and future prospects associated with the subject interest.*

History & Organization

This section provides many of the facts on which the valuation is based. You will want to provide a brief history of the entity and its property ownership, an organization chart if the entity organization is complex, and a short statement about the current state of any real property (leased, stabilized, rents above/below market, vacancy issues, highest & best use). Past sales of any interests in the same entity—or even other entities controlled by the same parties—can be discussed here.

An example of this section can be found in **Case 1 Illustration 3.1, pp. 66–69** with some overview text on **pp. 62–64**.

Agreements

This section should provide a detailed summary of the relevant LLC operating agreements or partnership agreements, any common tenancy agreement, or other agreements affecting rights that might impact entity operations. See the relevant section of **Case 1 Illustration 3.1, pp. 69–71** and the related overview text on **pp. 64–65**.

Value Influences

Suggested report text:

> *The organization structure presented in the Agreements section acts to separate the subject interest from the principal real estate holdings. The interest's ability to influence property operations is more limited than it would be if that interest held the property without partners, and its ability to exit the investment (particularly with its pro rata share of underlying net asset value) is restricted by agreement. This section is concerned with evaluating control and other elements that define the extent of the impairments, whose effect on value will then be determined in § III and § IV of this report.*

Although not shown in the book, it is a really good idea to include a summary of value influences after the agreements section, which serves as a useful place to summarize who has control, the extent to which the subject interest-holder can influence anything, how it can exit, and how the interest can be assigned or transferred. (It is helpful to include a short discussion about asset diversification, leverage, and distribution trends here, although that can come later.) Most of this information will have to come from your client in some form.

The PVX Dashboard questions that will guide you to consider the value influences are **Control/Mgt questions 4** and **5** regarding management and all **Control/Subject questions** regarding the subject interest's degree of control and its source.

Property and Market Analysis

Suggested report text:

The Property is described in the Appraisal (see Appendix F), and its major characteristics are summarized below. The market analysis that follows outlines key external economic and financial conditions. Both discussions are abbreviated, as we are also relying on the more detailed local market analysis presented in the Appraisal.

Asset/Real property information

The property overview summarizes basic details about the underlying real estate. You will probably have a real estate appraisal (separate or in your report's appendix), but it will help the reader's understanding greatly if you summarize elements of the appraisal that would be important to the partners. For example, you might discuss the nature of the property, whether it is owner occupied or leased (if leased, be sure to include the remaining term), whether it is stabilized or in some other state, any major issues, the concluded highest & best use, and whether that use might change in the future (say, 10–15 years).

A sample collection of pertinent information is shown in **Case 1 Illustration 4.1 on p. 78** and further analysis is shown in **Case 1 Illustration 4.3, p. 80**. A sample overview is shown in **Case 1 Illustration 4.10, on p. 100**. The information needed for the overview is gathered in the Dashboard's **Real Estate section, questions 1–3**.

Economic & market conditions

Suggested report text:

This section typically concerns the national economy, capital markets, and both national and local real estate market conditions, depending on relevance. In this case, local market dynamics are discussed in detail in the Appraisal. Applicable conditions needed for our analysis will be extracted from the Appraisal where available and supplemented with longer-term real estate trend data in this section.

Information under this heading can vary greatly, depending on what has been provided in the real estate appraisal. This is the time to look ahead at what will be needed for the valuation models. If the real estate appraisal uses a discounted cash flow model, then the appraisal report may provide everything you need. Otherwise, you may need to supplement the appraisal. See **Real Estate question 4** (and the Overview) and **Case 1 Illustration 4.2 on p. 79**. I usually include long-term GDP and CPI or PPI forecasts here anyway, using longer-term data from the Livingston Survey, for example. If you get economic information and growth rate data from the appraisal, then it is helpful to summarize it here. Make sure all variables used in the valuation sections of the report are supported.

Financial Statement Analysis

Suggested report text:

Financial records reviewed for this analysis include management's operating statements for the years ending December 31, 20XX through 20XX. The purpose of our analysis in this section is to present a balance sheet as of the date of value, normalize and adjust that balance sheet for

our valuation purpose, and develop a pro forma of property operations and a statement of cash flows expected for 12 months following that date. This section begins with [the Entity]'s assets and liabilities.

Balance sheets

You will need to spread historic financial statements (balance sheets and income statements) for the operating entity for *at least* the three years just prior to the date of value. We prefer five years, although an important event like a past recession that has affected operations might call for an even longer period. The income statements are the same source that is used in the real estate appraisal. The spread financials should be included in the report appendix.

A financials overview is provided on **pp. 104–105**. The balance sheet is discussed in **Case 1 Illustration 5.2, p. 108**, and in **Entity questions 1–5**. The income statement begins with the real property statement from the real estate appraisal, which is then normalized for its operation by the subject entity; see **Case 1 Illustration 4.7 on p. 93 and 4.8 on p. 95** and **Real Estate questions 5–6**.

Common tenancy is quite similar, but the client will not necessarily have financial statements that are as well organized as they would be for an LLC, for example. The differences are spelled out with an overview on **pp. 230–233**; see also, **Case 2 Illustration 9.2 on p. 232**.

Cash flows

Entity cash flows are next, with overview text on **pp. 107–113** and more detailed information in **Case 1 Illustration 5.3 on p. 111 and 5.4 on p. 114**. See also, **Entity question 6**.

For common tenancy, you can read more on **pp. 233–238** and in **Case 2 Illustration 9.3 on p. 234**.

Restriction (Holding) Period Analysis

Suggested report text:

An understanding of the hypothetical buyer's restriction period expectations is needed for evaluating marketability/liquidity impairment (i.e., how long the buyer will be "trapped" in the investment), in § III and IV. The likely restriction period is not fixed but is developed by considering the facts and circumstances relating to the real property and the interest-holder's options. We will consider both the investment period that might be the objective of a buyer of the interest, and the restricted period during which the buyer would not be able to respond to changing market conditions.

This is the last section before the valuation process and in many respects the most important. Value is almost always sensitive to time. In this context, that is the period during which the subject interest-holder is unable to exit its position. At the end of that period, it either receives, or has the right to receive, its pro rata share of the entity's net asset value. See overview and **Case 1 Illustration 5.1 on p. 103**, or **Case 2 Illustration 10.1 on p. 249** for common tenancy. All **Restriction Period/Time questions 1–10** apply.

III. Partnership/Company Valuation Analysis

Now we are at the point in the report where concepts may be less familiar to the reader, so we need more complete explanations.

Overview

Suggested report text:

The bundle of rights associated with a minority interest in a business is considerably reduced from those associated with ownership of the whole. Potential buyers "discount" the pro rata value of such interests, because of various factors which limit their ability to control the enterprise or the assets or dispose of the interest. The most important limitations are lack of control and lack of marketability. These limiting conditions increase the yield required by investors for all types of investments, and discounting price increases yield. Therefore, limitations on property rights may be considered by analyzing either discount or required yields; they are two sides of the same coin.

The first idea the reader needs to understand is that the holder of the entire property has certain rights that are progressively stripped away with any form of shared ownership, and that reduced rights lead to increased risk and discounts.

Begin with the "levels of value" idea from **pp. 27–30**; there are two different versions, one for Partnership/LLC valuations (left diagram in **Figure 2.2**) and another for common tenancy (right diagram). Common tenancy is detailed further on **pp. 241–248**. Consider including the **Figure 10.2** decision tree.

Insert the appropriate levels figure and adopt as much text as you like from the levels of value discussions in chapters 2 and 10. For common tenancy, it is worth considering a description of how partition fits into the analysis, as is described on **pp. 245–247**. It is imperative that the valuer be able to articulate the core principles underlying the entire value analysis simply and cleanly when needed (say, in your direct testimony).
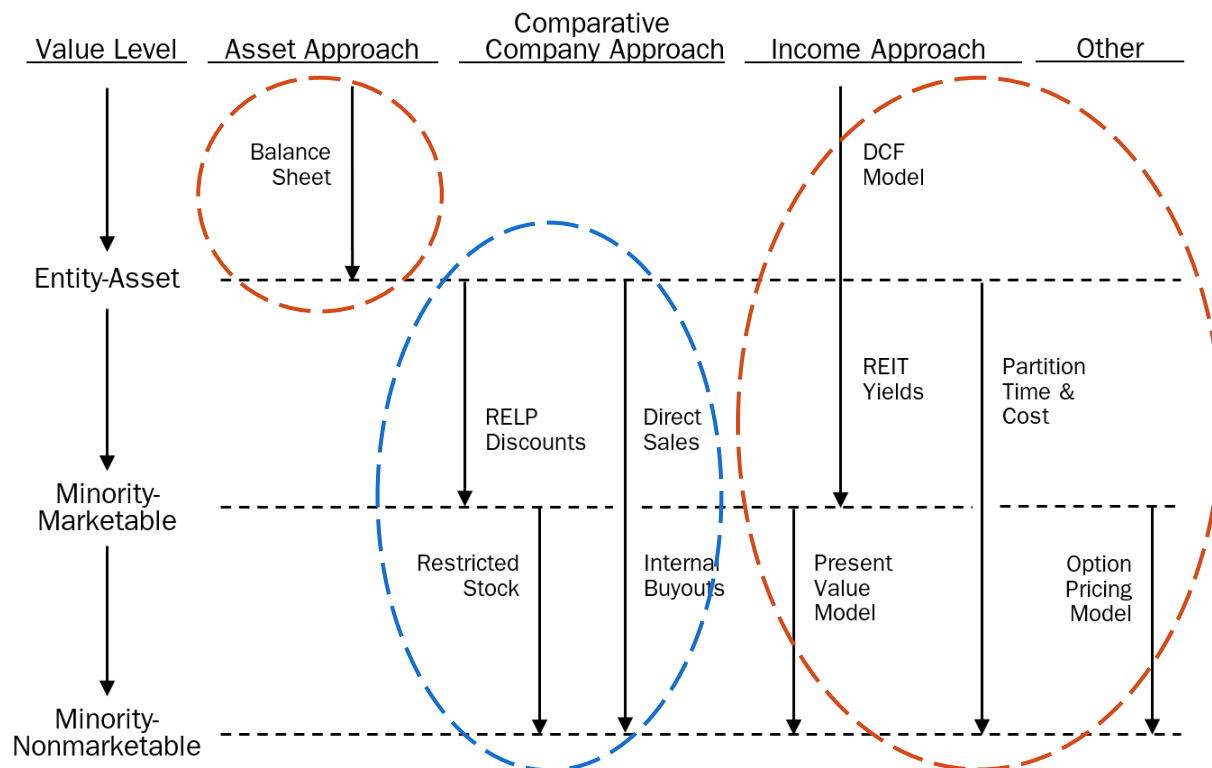
Valuation Approaches

This is comparable to the Approaches section of any standard appraisal report but constructed primarily around the business valuation process. It begins with Revenue Ruling 59-60 from **pp. 40–43**—generally considered the starting point—where you will explain how your valuation process satisfies each of its requirements. You can also include Revenue Ruling 77-287 on **p. 43**. Neither is binding, and the references might not be meaningful outside the United States, in which case you can still state the principles of the two rulings and show how the list of conditions was satisfied. You can insert the descriptions of both Revenue Rulings from the pages listed above.

The Valuation Approaches section provides details of the valuation methods that were mentioned earlier, in your scope of work statement, but in this case you must clearly and completely describe *all* the possible methods, stating why you used the ones you selected and why you rejected the others. See **pp. 55–62**.

It is likely that you will use more than one method, as shown in the figure below.

Value Level	Asset Approach	Comparative Company Approach	Income Approach	Other
Entity-Asset	Balance Sheet		DCF Model	
Minority-Marketable		RELP Discounts / Direct Sales	REIT Yields	Partition Time & Cost
Minority-Nonmarketable		Restricted Stock / Internal Buyouts	Present Value Model	Option Pricing Model

You might consider including something similar in your report so that the reader can easily see which of the three primary approaches was used at which of the three levels of value. PVX uses the net asset method at the asset level as a reference point, then the income approach/DCF method, which includes the asset level and leads to both the minority-marketable and minority-nonmarketable levels. The partition model also belongs in the income approach. You can include other methods as checks of reasonableness, such as the Partnership Profiles discount study, which belongs in the comparative company approach. This is the place to cover all relevant methodology.

Asset accumulation approach, NAV method

Suggested report text:

> The Company is indeed an asset holding company. Property operations will very likely continue in their present form for an indefinite period. Operations are stable, so the NAV method is both conventional and reliable for our purpose. We use this method to value the Company as a whole; subordinate methods of determining applicable discounts are discussed below.

See pp. 55–56.

Comparative company approach, various methods

Suggested report text:

This method is not used directly for developing the value of the Company as a whole, because its economic circumstances are best represented by the real estate asset, which is analyzed as part of the real estate appraisal process. The marketability studies are included as a test of reasonableness for the discount for lack of marketability that is determined using a present value method.

Another comparative company method is often used to develop discounts from NAV to minority level value, using trading data and calculated discounts observed in RELP transactions. These data are extremely difficult to apply to the subject Company, and especially for the modified management risk that the Company represents. Accordingly, the RELP discount analysis would be grossly unreliable, and is not used in this report.

See pp. 56–58.

Income approach, DCF and present value models

Suggested report text:

Income methods will not be used directly to determine the value of the Company as a whole. However, we do use a DCF model to measure asset-level yield and growth rates for cash flow and value. These values are then adjusted for the minority position by adding risk premiums for impaired control and related factors. Lastly, a present value model is used for analyzing the adjusted risk associated with the subject interest during its restriction period.

See pp. 58–59.

Other methods [if applicable]

Suggested report text:

An option pricing model is used together with the present value model to determine value at the minority-nonmarketable level of value, since the anticipated restriction is very short, and short periods must be analyzed using the two methods.

Income Approach – Discounted Cash Flow Method

Increasing the asset-level yield rate is (of course) part of the income approach; in PVX, this is the discounted future returns method. The first step is to describe how the income approach works, which is covered in chapter 11. An overview is provided on **pp. 292–296**; although very little of that is needed in the report, you should certainly understand it and convey to the reader that 1) yield buildup is a conventional process, and 2) it is parallel to discounts, as shown in levels of value figures.

Capitalization/yield rates

The overview for this section of your report can begin with a discussion of capitalization/yield rates that can be taken directly from **pp. 297–302**. (The method descriptions on **pp. 303–306** include those methods that PVX can handle—shown in **Figures 11.2** and **11.3**—and others that

it cannot handle without additional workarounds. These are only for your information and are not normally included in the report.)

Asset-level yield

Suggested report text:

> *Risk to the entire Company, or to an interest-holder that controls the Company and its real estate assets, is very closely tied to the real estate itself. But that risk can be altered by the presence of any other assets and liabilities, most prominently any mortgage financing. We will measure this overall Company risk by using a 10-year DCF model to project annual cash flows over the term and the proceeds of a hypothetical sale of the underlying real estate and any other balance sheet items (such as any mortgage balance remaining) at the end of the term.*

The asset level is the yield rate starting point for control discount analysis in the next section. The basic DCF model is discussed on **pp. 113–115** (for LPs and LLCs) and **pp. 235–238** (for common tenancy). This model gives you two ways to find the yield rate:

1. For income-producing properties, you simply input the capitalization rate and the 10-year growth rate from the real estate appraisal, the appraiser, or from other sources, as described in **Real Estate questions 5 and 6**. PVX automatically creates a 10-year DCF table in the Valuation Models section that calculates the entity/asset-level yield rate and the value and cash flow growth rates that are shown in **Entity Advanced question 7** and explained in the Yield and Growth Analysis section of the Discounted Cash Flow Model in your UVS. The DCF in PVX is similar to the ones shown in **Case 1 Illustrations 5.4 on p. 114 and 11.1 on p. 306 and Case 2 Illustration 9.4 on p. 236**.
2. For non income-producing properties, such as undeveloped land, you will zero cash flows in the **Entity** window (as explained in **question 6**), which causes yields to be determined differently. Instead of inputting the appraisal's capitalization rate (since it will likely not have used an income method in the first place), you will input a cap rate based on the asset type. PVX provides default rates based on your "create new" project entries, but it would be best to work out actual cap/risk rates (say, for example, for a ground lease rate for undeveloped land based on its potential use) with the real estate appraiser. PVX calculates the yield rate as the sum of the cap rate and growth rate, as described on **pp. 301–302** and in the second part of **Case 1 Illustration 11.1 on p. 307**.

The DCF and its elements are described in the Valuation Models section of the UVS (if you are looking at an LLC/LP project, you can just skip the minority-level parts). You can copy the entire DCF model into the body of the report, or place it in appendix B. You can also use descriptions like the ones in **Case Illustrations 5.4**, **9.4** and **11.1**. You can then summarize results using the formats shown in **Case 1 Illustrations 5.5 on p. 117 and 11.1 on p. 307 and Case 2 Illustration 9.5 on p. 238**.

IV. Subject Interest Valuation Analysis

Suggested report text:

> *The previously concluded net asset value and yield rate assumes full control of the Company and its assets. However, the subject interest-holder does not necessarily have such control, nor can it readily exit its position, both as described in the value influences summary. We will now analyze these impairments, which require adjustments to the yield rate.*

Minority-Level Value Analysis

Now the fun begins. You have built an asset-level foundation and have determined yield and growth rates appropriate to this level. The remainder of your analysis is actually quite simple: building up from the asset-level yield rate to the nonmarketable-level yield risk by accounting for control impairment generally, then for specific risk elements that you addressed in the Control and Restriction Period Risk sections of your Dashboard. It's all ready for you to summarize in your report.

Entity-related concerns

The first yield adjustment is for almost complete loss of control, as represented by an analysis of REIT trading, described on **pp. 148–161**. This is a long section that uses data through 2018 for illustration; the REIT data are shown in **Appendix C**. PVX updates these data each year and provides an unadjusted REIT yield premium (Y_F, see **UVS: Control Subject Calculations, Management Risk section**) based on the date of value. You can summarize the process in the body of your report, place a long summary in your report's appendix (preferred), or simply footnote the relevant passages in the book.

The REIT data reflect control that is exercisable by the public shareholders (which is essentially none, although they can still exit at the share's minority value). REITs also have professional management. However, the degree of control and level of management ability are likely to be different for your subject entity. Its management may not be as competent or have the same level of confidence, and succession might be a riskier prospect than for typical REIT management.

LLCs and LPs
The Management Risk Classification System (MRCS) is described in detail on **pp. 131–134** and in the **PVX Control MGT window Overview**. You only need a short introduction and can cite **pp. 131–134** and the adjustments on **pp. 161–163**. You may also wish to include the risk level classification depicted in **Figure 6.4 on p. 162**. **Case 1 Illustration 6.4 on p. 164** has most of the text you will need to report your results. Your actual results are shown in the **UVS Control (Subject) Calculations, Management Risk section**.

Common Tenancy
The yield rate adjustments for control are developed in the same manner as above, but with slight variations because of the unstructured nature of common tenancy (and the often unusual structure of general partnerships). You will want to review the MRCS overview on **pp. 254–256**. The adjustments are the same as those for partnerships on **pp. 161–163**. You may also wish to include the risk level classification chart depicted in **Figure 6.4 on p. 162**. **Case 2 Illustration**

10.3 on p. 260 has most of the text you will need to report your results, although this illustration is more complicated than necessary at this point. As with LLCs and LPs above, your actual results are shown in the **UVS Control (Subject) Calculations, Management Risk section**.

Subject influence

The analysis of subject interest control attributes is similar to the MRCS above, although in this case the yield rate is reduced as subject control is increased. In the **UVS Control (Subject) Calculations, Subject Risk section**, PVX adjusts the REIT yield premium Y_S to get Y_F.

LLCs and LPs
The control attributes overview for LLCs and LPs is on **pp. 168–173**. The degree classification chart is depicted in **Figure 6.5 on p. 170**. **Case 1 Illustration 6.6 on p. 173** has most of the text you will need to report your results. Your actual results are shown in the **UVS Control (Subject) Calculations, Subject Risk section** and the **Restriction Period Properties (Risk) Calculations**. This is also an element you can use to account for intangible benefits of ownership, **p. 173**.

Common Tenancy
Common tenancy is discussed here but will be applied a little differently, as seen in the Present Value Model section below. Presenting your analysis of subject interest control attributes is similar to the management analysis, although in this case the yield rate is reduced as subject control is increased. The control attributes overview for common tenancies is on **pp. 256–259**. The degree classification chart for common tenancy is shown as **Figure 10.3 on p. 259**. **Case 2 Illustration 10.4 on p. 260** has most of the text you will need to report your results. Your actual results are shown in the **UVS Control (Subject) Calculations, Subject Risk section** and the **Restriction Period Properties (Risk) Calculations**. This is also an element you can use to account for intangible benefits of ownership, **p. 284**.

Minority-marketable value and discount

For structured entities like LPs and LLCs, PVX provides calculations for minority-marketable value and a discount from NAV, as both are generally expected for structured entities. Minority-level discount calculations are shown in the **UVS DCF Model's Minority-level Rates section**, where the minority-level yield rate (Y_C), growth rate (G_{VCN}), and distribution rate (R_{CM}) are calculated for use in the **Present Value Model's discount for lack of marketability**. The minority-level value is shown at the end of the DCF Model and included in the **UVS DCF table.** The discount is shown for disclosure, but it plays no role in the rest of the analysis.

The whole-entity minority value or the discount are useful in situations where other comparisons are being made based on market capitalization (the securities trading term for whole-company minority-marketable value), or discounts taken from public partnership trading. You can use the discount to compare results with other sources of minority values or discounts, but you should be mindful of the adjustments to the REIT yield that have been made (for management and subject control) and make sure those adjustments are consistent with the other data sources. You can make adjustments for this purpose (reduce subject control to the 1st Degree and management control to an appropriate level) as a side calculation. This is useful for a reasonableness test only, and not necessary for your report.

The minority-marketable value is not meaningful for common tenancy interests, as described in the levels of value discussion on **pp. 241–242**, since complete loss of control is highly unlikely.

Nonmarketable-Level Value Analysis

Suggested report text:

We have established that risk increases in relation to ownership of the whole property (the asset level) when an interest-holder's control is impaired. The consequences of control loss apply even if the interest were fully marketable. However, the interest is not readily marketable for the duration of the restriction period. This exposes the subject interest to a variety of additional risks, none of which can be responded to. We refer to the increased risk as the Investor's Required Return (IRR).

A short introduction to the marketability discounting process appears on **p. 178**. This is a separate step for structured entities, but not for common tenancy, which considers control loss as just one element among the others considered. Rather than creating a separate section, it would be easier to first make the above control adjustments, then continue with the additional yield rate adjustments for restriction period risk.

The concluded restriction period might be the same as the entity DCF's 10 years but could be anywhere from 4 to 15 years (for LLCs and LPs) or 6 months to 15 years (for common tenancies). To accommodate the variability of this period, PVX uses a present value model like the one on **pp. 203–205**.

Present value model

LLCs and LPs
Income methods are described beginning on **p. 190**. It is a good idea to describe the process generally, using **Figure 7.2 on p. 203** if you wish. The present value model for LLCs and LPs begins at the minority-marketable level and ends when the interest-holder receives (or has the right to receive) its pro rata share of undiscounted NAV at the end of the restriction period. One of its trickier aspects (and one that is often missed) is that growth rates are increased because the initial value has been decreased from the asset level (NAV) to the discounted minority-marketable level. A lower starting point with the same end point means a greater value growth rate. The analysis is fairly long; rate changes are discussed on **pp. 192–195**. Fortunately, all you need to do is report these minority-level rates (and calculations if you wish) from the heading of the same name in the **UVS Discounted Cash Flow Model**.

The yield Y_A was adjusted to account for control loss under the DCF Minority-level Rates heading. Now Y_C will be adjusted upward again to account for risks you have identified in Restriction Properties, to get the investor's required rate, Y_I; see the **UVS Restriction Period Properties (Risk) Calculations section**. You can take descriptions directly from your notes in the UVS, along with the actual percentage adjustments, and report your results using the format shown in **Case 1 Illustration 7.3 on p. 198**.

Y_I and the other rates will be used in the present value model calculations, which are shown in the **UVS Present Value Model**. The calculations conclude the discount for lack of marketability D_M, which is then combined with the discount for lack of control to give the overall discount

D_{PTR}, which is the discount displayed on the Dashboard. The present value model calculations are shown in **Case 1 Illustration 7.5 on p. 204**, as well as in the **UVS Present Value Model**. Discount arithmetic is discussed on **pp. 203–210**, where process descriptions are also provided for your adaptation.

General Partnerships
A general partnership (GP) is a hybrid type of entity, because it frequently offers a means of exit that is not available for most limited partnerships. GPs are discussed on **p. 26**.

A limited partnership (LP) is expected to "trap" the subject interest-holder for a longer period, so the restriction period setting is limited to four years or more. If a GP ends up with a restriction period of four years or more, then it should be valued as if it were an LP. Conversely, if an LP ends up with a restriction period of less than four years, it should be valued as if it were a GP.

Common tenancy with an operating agreement can be somewhat restrictive and require valuation as a GP, or very restrictive and require valuation as an LP.

The GP setting operates like common tenancy in that the minority-marketable level of value has little meaning, so it is ignored in the valuation process. The present value analysis is the same for GPs and common tenancy.

Common Tenancy and General Partnerships
Common tenancies and GPs do not use a minority level for analysis since comparison with public partnerships is all but meaningless; see **pp. 27–30**. Control-related risks are analyzed in the same way as for LPs, as described under the Minority-Level Value Analysis heading, above. Unlike the LP analysis, the common tenancy/GP analysis does not conclude a minority value before proceeding with the Nonmarketable-Level Value Analysis. Control-level risk is now included with restriction-period risks.

Income methods for common tenancy are described beginning on **p. 252**. It is a good idea to describe the process generally, using **Figure 7.2** if you wish. Unlike the model above, this model begins at the asset, not the minority-marketable level; it still ends when the interest-holder receives (or has the right to receive) its pro rata share of undiscounted NAV at the end of the restriction period. Growth rate analysis is not as complicated because there is no minority level of value; the growth and other rates are the same as at the asset level and are discussed on **pp. 262–263**. They are reported in the **UVS Discounted Cash Flow Model and DCF.**

The remaining element is risk adjustment, this time adjusting from Y_A directly to Y_{AC}, to account for risks you have identified in the **Restriction Period (Risk) window**. You can take descriptions directly from your notes in the UVS, along with the actual percentage adjustments shown in the **UVS Restriction Period Properties (Risk) calculations**.

The present value model calculations are shown in **Case 2 Illustration 10.5 on p. 265**, and in the **UVS Present Value Model**. Discount arithmetic is discussed on **pp. 203–210** and **pp. 265-267**, where process descriptions are also provided for your adaptation. The calculations conclude a combined control and marketability discount by this method as D_{PV}. Depending on the circumstances of your valuation, it may have to be reconciled with discounts from one or two other models, below.

Black-Scholes model

Common Tenancy

PVX will also use the Black-Scholes model if the restriction period is shorter than four years. The entire theory and development of the model is set forth in chapter 12. The book does not have an easy overview in this case, so you may wish to use the text provided in the supplement to this report guidance. You can certainly place book citations in your footnotes.

This is the most complicated model and the most difficult to explain, but it is also not often used. Some text is provided here, and more detail is included under the Appendix heading, below, to help. You can also use lots of footnotes. PVX will give you all of the numbers, so your report can readily support the requirement that another valuer could reproduce your results.

Suggested report text:

The difficulty of capturing short-term market behavior is a major shortcoming of the income approach. This limitation can be solved, however, by adapting the Black-Scholes Option Pricing Model, which is a good method for modeling short-term impairments. The basic idea is that a put option allows the holder to sell specified shares, at a specified price and at a particular time in the future. If one holds shares that are not directly marketable and purchases a put option, then the holder has effectively purchased the shares' marketability.

The Black-Scholes put formula is somewhat complex, but it basically shows a discount from market price (cost of the put divided by the price of the stock) that varies directly with time and the volatility of the underlying stock, and inversely with the prevailing (risk-free) interest rate.

The model is adapted for our purpose by adjusting its volatility term so that its indicated discount matches the present value model at four years. It is given increasing weight as time shortens [FN **Figure 12.2a on p. 323** and **pp. 320–327**]. The Black-Scholes model is described in detail in Appendix E.

The Black-Scholes formulas and calculations from **pp. 328–329** should be included in your appendix at least, with a calculation description from **pp. 329–330**, edited to fit your case details, in the body of the report. The calculations that result in volatility term selection, **Case 2a Illustration 12.1 on p. 330**, are a bit over the top, and can be left out (with a citation for this book example, if you wish). The rest of the calculations, **Case 2a Illustration 12.2 on p. 332**, can be taken directly from the **Black-Scholes model** in the Valuation Models section of your UVS and inserted in your report appendix. It concludes a discount (D_{BS}), which is reconciled with the present value model discount (D_{PV}) in the **UVS Reconciliation** section, see **Case 2a Illustration 12.3 on p. 334**. As shown in the diagram below, there may be an additional step of reconciling with the partition model discount, if applicable.

Partition model

Common Tenancy

A partition analysis is almost always expected for common tenancy interests, so it should be considered and presented in the report even if it ends up being meaningless (because no one in their right mind would bring such an action against the other owners, or because it is not economically feasible, for example). Economic feasibility is shown by the Partition Feasibility meter in the Partition Window.

The partition time and cost method is discussed on **pp. 245–247 and 267–274**. You can take a short overview from **pp. 267–268**, but it would be good to include the entire enumerated portion from **pp. 268–270**. The **PVX Partition DCF** is the same DCF model shown in **Case 2 Illustration 10.8 on p. 271** and the partition model calculations are shown in the **Valuation Models section**. You will also need yield rate buildup (similar to the one you used for the present value model) from the case illustration. You can find the buildup calculations in the **UVS Partition Properties Calculations**. This is an important model that is often misused, so take extra care with its presentation in your report.

Subject Interest Value Conclusion (with Reconciliation)

The LLC/LP analysis uses only one model for control and one for marketability, unless you have added supplemental methods that would either test the reasonableness of the two discounts or provide additional control or marketability discounting methods that would have been reconciled in those sections. This last heading will just summarize the overall process, as shown in **Case 1 Illustration 7.9 on p. 211**. The respective values are provided in the **UVS Conclusion**, although if you have used other sources and changed the discount(s), your final calculations will have to be done offline.

The common tenancy analysis uses two models (present value and partition) most of the time, and it adds Black–Scholes for short restriction periods. The **UVS reconciliation section** shows only the models that have been used, their discounts, and their PVX weightings, following the process that is described for general partnerships and common tenancy in the flow diagram below. If you have added other models, such as transactional data, your final calculations will again have to be done offline.
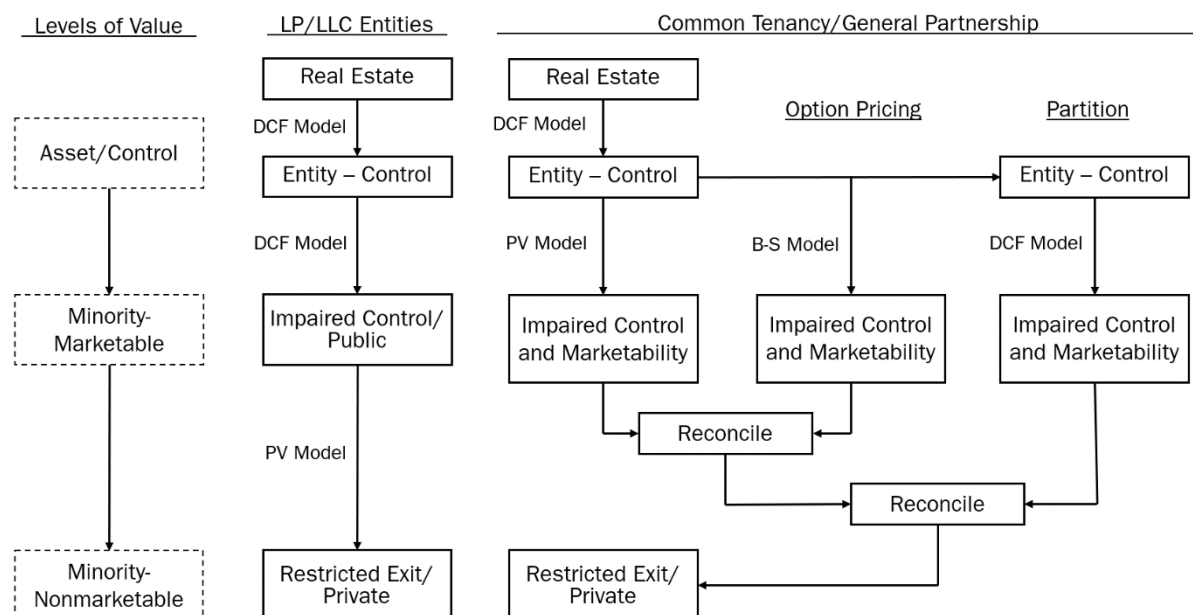
(If you have mentioned not using sales in the scope of work section, including a description from **pp. 274–282** in the appendix can help explain exactly why transactional data is not included; that way, you can easily rebut the other side if they do use transactions.)

The conclusions are summarized in **Case 2 Illustration 10.11 on p. 286**.

Please follow the conclusion with a reconciliation to prices paid in the past for interests in the same entity as the subject, if any, as described under **Historical Sales on p. 285** and in **Case 2 Illustration 10.11**. You can also put in a sensitivity analysis if you wish. Comments concerning the hypothetical seller are also a worthwhile addition, particularly if the facts have led to a large concluded discount.

Certification

This refers to the certification of the fractional interest valuation. The real estate appraisal has its own certification, although they could be combined if you wish.

Appendices

Fractional interest valuations cover a lot of territory and require supporting documentation. They can also include long explanations. You can make it easier on the reader if you include reference materials that would otherwise interrupt the flow of your "story of value" in the appendices of the report.

Documents

Include any operating, management or other documents that you relied on for your analysis. Common tenancy may have only deeds, although these are not always required.

Financials

Client financial history should be provided as received, although it is helpful for both you and the reader to spread them by year rather than including the originals. You can also include any financial analysis you have generated but that may be too extensive for the body of the report, including DCF models and calculations.

Real property data

Real estate documents are normally included as part of the appraisal, although if they are not, and if you relied on information that was not clearly reflected in the real estate appraisal, you can include them here as well. You can even include entire real estate appraisals or mix in real estate data if you have combined the real estate appraisal and the fractional interest valuation in one report.

REIT market yield data

The explanation of REIT risk premium development is a long one, so you can either footnote the book section or include in the Appendix as a separate document. You can also replace the 2018 tables with those appropriate for your date of value.

Black-Scholes model

Extract from chapter 12 text and include as a separate document. Include formulas and calculations from the UVS.

Supplemental methods

You can footnote relevant marketability studies from the book or include as a separate document. Not needed unless you are relying on value indications from the studies, or from other valuation tools such as the Stout Restricted Stock Study & DLOM Calculator.

Public Limited Partnership Secondary Market discount studies.

Appraiser qualifications

Include your CV and any temporary practice permits.

Real estate appraisal(s)

Can attach or incorporate by reference; this is an integral part of your report.

Report Preparation Guide Outline

VALUATION REPORT OUTLINE	BOOK REFERENCES	PVX REFERENCES

Title Page
Letter of Transmittal
Executive Summary
Limiting conditions

I. Introduction

    A. Subject Ownership Interest

    B. Purpose & Use

    C. Date(s) of Value

The <u>front matter</u> is not modeled in the book or PVX and should follow your desired format.

The <u>subject of the report</u> is the interest (or interests) being valued.

The <u>executive summary</u> is meant to highlight basic analysis and conclusions so the reader can understand the scope of the valuation, but with limited explanation. It is not absolutely necessary, but is usually very helpful for the reader, especially for longer reports.

<u>Limiting conditions</u> are generally for business valuation (conforming with USPAP Standard Rule 10, for example), but will also include conditions applicable to the real estate if you have combined the real estate appraisal in this report. Otherwise, real estate limiting conditions belong in the real estate appraisal document.

Items I.A–I.C are very specific to each valuation and should also follow your desired format.

See introductory detail in Case 1 Illustration 3.1, p. 66.

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II. Entity, Property & Economic Conditions			
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B. Agreements	pp. 64–65 pp. 69	overview Case 1 Illustration 3.1	
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C. Property & Market Analysis			
Asset/Real Property Information	p. 78 p. 80 p. 100	Case 1 Illustration 4.1 Case 1 Illustration 4.3 Case 1 Illustration 4.10	• Real Estate, Qs 1–3
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VALUATION REPORT OUTLINE	BOOK REFERENCES		PVX REFERENCES
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Cash Flows	pp. 107–113 p. 111 p. 114 pp. 233–238 p. 234	overview (LLCs/LPs) Case 1 Illustration 5.3 Case 1 Illustration 5.4 overview (CT) Case 2 Illustration 9.3	• Real Estate, Qs 5–6 • Entity, Q 6
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C. Income approach: DCF Method	pp. 292–296	overview	• UVS: DCF Model • UVS: Discounted Cash Flow Table
Capitalization/Yield Rates	pp. 297–302 pp. 303–306	overview examples	
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APPENDICES			
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D. REIT Market Yield Data	pp. 150–155	or separate document	• Tables 6.3 and 6.5 with date of value REIT statistics (provided in UVS for full subscribers only)
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Supplement to Report Preparation Guide

<mark>Yellow highlight</mark> in the samples below indicates text and values to be deleted or replaced with your own in your final product. Style and numbering of suggested footnotes should be adjusted to match your own report style.

REIT Market Yield Data Appendix [SAMPLE]

It has been demonstrated in the body of this report that the subject interest holder's rights are impaired because of its lack of control of the underlying real estate, and that the value of its interest should be reduced accordingly. We have also noted that reducing value increases yield, and the difference between the lower-risk yield at the asset level and the greater-risk yield at the subject interest level is the *yield premium* that compensates for the interest holder's impaired control. Although the yield premium is not necessarily related to owners' equity, applying either a yield premium or a discount will result in a reduced subject interest value. Accordingly, an effective discount can be calculated for a yield premium-based analysis as:

discount = 1 – subject interest value / owners' equity

Discounts and yields are essentially two sides of the same coin. Our analysis of subject interest risk is based on yields, as shown in § III-D, Undivided Interest Risk. The control element used in that risk analysis is extracted from the REIT market yield data that is presented in this Appendix C.

REIT trading provides an extremely useful body of data from which we can extract the needed yield premium. This provides a coherent and fundamental basis for analysis, since any financial investment is fundamentally about returns (yield).

REIT Market Data and Real Estate Returns

REIT trading provides an extremely useful body of data from which we can extract the needed yield premium. Such data are ideal for risk premium analysis because the market operates with full knowledge of total returns and other related metrics—REITs are creatures of yield, and total yield is readily available to investors. The REIT market is huge, and most REIT shares are exchange-traded. The data are published and analyzed by NAREIT, [1] and are readily available to the valuer.

Market-observed yields for REITs reflect both the real estate and its holding entity. We are seeking not total yields, but yield premiums from these market data, and can do this by comparing REIT (minority-level) yields to real estate (asset-level) yields. The comparison we are seeking is the same as the difference between the asset- and minority-levels of value: the difference between returns required by fee-holding real estate investors and returns demanded by minority shareholders in the REIT markets. The difference between the fee and non-controlling yields would then be the control risk premium.

REIT shares exhibit significant price volatility, though, and it has not been clear how to extract needed yield data from REITs. If control is a systematic variable, then it should be revealed if we use observations over a long enough period.

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[1] The National Association of Real Estate Investment Trusts, https://www.reit.com/data-research/reit-industry-data.

Data[2] are published by NAREIT for 226 REITs, 186 of which are traded on the NYSE. The equity market capitalization[3] is $0.97 trillion, and the REITs together hold $2.0 trillion of commercial real estate assets. NAREIT also publishes an Equity REIT Index,[4] which is often used to demonstrate long-term equity returns.[5]

Real estate market survey data are also available from the PwC Real Estate Investor Survey.[6] The PwC Survey concerns primarily institutional real estate,[7] and with its predecessor, the Korpacz Real Estate Investor Survey, has been published in a consistent manner since 1988. The PwC Survey reports on the principal investment property types, which generally match property types held by REITs. The PwC Survey also computes a broad yield measure, the PwC Yield Indicator (PYI).[8] The PwC Survey is used by commercial real estate professionals as an authoritative standard for institutional property returns.

REIT data for this analysis were taken from NAREIT's REITWatch publications for January 1999–2018 (calendar years 2000–2019). The data represent 213 different REITs and 1,710 REIT/year combinations.

REIT data show considerable volatility year-to-year, while the PwC real estate returns appear to be more stable. Each varies as a function of different external and market conditions, although the PwC data are expected returns, so they appear more stable. Annual matching is not useful, but looking at multiple periods is. In particular, analysis across at least an entire real estate cycle[9] would be expected to smooth out nonsystematic (short-term) variations and heighten the more fundamental systematic variations. As expected, analysis of longer periods results in less variation in returns, as shown in Table C-1.

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[2] The REIT Narrative Data table (3rd table) in the REIT Data Tables section of your User Valuation Summary contains the information you need to replace the relevant highlighted values in the text. The table does not need to be pasted into your report.

[3] NAREIT, REITWatch, (January 2019). Equity Market Capitalization is defined by NAREIT as "Price on the date indicated times the number of common shares outstanding."

[4] Annual Returns for the FTSE NAREIT U.S. Real Estate Index Series. This index includes all Equity REITs not designated as Timber REITs. REITs hold assets in the residential, industrial/office, retail, lodging/resorts, health care and self-storage sectors, and some are diversified or are hybrids. Sixty percent are in the residential, industrial/office, and retail sectors.

[5] Equity returns are measured as total returns to equity. NAREIT calculates total returns by taking the closing price for the current period, adding any dividends with an ex-dividend date in that period, then subtracting the closing price for the previous period and dividing the result by the closing price of the prior period (see NAREIT, REITWatch, supra).

[6] PwC LLC, PwC Real Estate Investor Survey (multiple years).

[7] See PwC Survey, supra. Institutional-grade real estate is defined as: "Real property investments that are sought out by institutional buyers and have the capacity to meet generally prevalent institutional investment criteria."

[8] The PwC Yield Indicator is defined as: "A composite IRR average of the surveyed markets excluding lodging and development land."

[9] An examination of the Real Estate Value Cycles sections of the PwC Survey from 1999 through 2019 determined that period had two cycles of about seven years each.

Table C-1

REIT Yields All Sectors

Period Ending	Rolling Average Returns			
	15	12	10	7 years
2005				21.6%
2006				26.6%
2007				20.4%
2008			16.2%	15.5%
2009			18.1%	16.2%
2010		17.6%	17.6%	14.9%
2011		18.0%	16.4%	11.4%
2012		16.9%	16.9%	12.2%
2013	16.0%	15.5%	13.4%	7.6%
2014	17.9%	16.7%	13.1%	13.0%
2015	16.0%	14.1%	12.3%	16.1%
2016	15.9%	12.9%	10.5%	15.8%
2017	15.4%	12.2%	12.1%	12.5%
2018	12.5%	9.0%	13.2%	11.0%
Average	15.6%	14.8%	14.5%	15.4%
CV	0.10	0.19	0.17	0.31

As expected, analysis of longer periods results in less variation in returns. The four columns show the maximum number of observations we can extract from 1999–2018 data. The coefficient of variation (CV)[10] for the six 15-year rolling average periods is much lower than for the shorter periods, as would be expected. Interestingly, these REIT returns look very much like published, marketability-adjusted returns for real estate limited partnerships.[11]

Using a 15-year rolling average of returns limits analysis to the six 15-year periods ending in the years 2013–2018.[12] Table C-2 shows 15-year rolling average returns for equity REITs, the PYI, and the implied premium, as:

premium = REIT – PYI.

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[10] CV = Standard Deviation / Mean. CV is a measure of data dispersion, and a smaller CV indicates that the data are more tightly grouped.

[11] Partnership Profiles, Inc., "Rate of Return Study: Publicly-Held Real Estate Limited Partnerships and Real Estate Investment Trusts." Available at: www.partnershipprofiles.com. The study is also excerpted in Johnson et al., *Comprehensive Guide for the Valuation of Family Limited Partnerships*, 5th ed. (Dallas: Partnership Profiles, 2017): 18–23. RELP returns published in the Rate of Return Study, adjusted for the effect of the trading market discount, are almost identical to the REIT returns examined in this section.

[12] Our detailed analysis is limited by the availability of published REITWatch data, which begins in 1999. A longer series would be nice, but long-term data show pretty much the same yields. The Index is constituted differently from the REITWatch data shown in the tables, so yields are lower, but yields are still quite consistent over time.

Table C-2

Unlevered Yield Premiums All Sectors

Period	15-Yr Rolling Average Returns			
Ending	REIT/	REIT/	PYI	Unlevered
	Equity	WACC		Premium
2013	16.0%	12.1%	9.8%	2.3%
2014	17.9%	13.3%	9.6%	3.7%
2015	16.0%	12.1%	9.4%	2.7%
2016	15.9%	12.0%	9.1%	2.9%
2017	15.4%	11.7%	8.9%	2.8%
2018	12.5%	9.8%	8.6%	1.2%
Average	15.6%	11.8%	9.2%	2.6%
CV	0.10	0.09	0.04	0.29

The WACC (weighted average cost of capital) column is the result of adjusting total equity returns for leverage used by REIT mangers. Such leverage is meant to increase returns (among other things), and it creates a distortion of the returns that must be removed to have a meaningful comparison with the PYI returns.[13]

The 2.6% average yield premium from Table C-2 is the conclusion of this analysis: an increase in the yield rate of 2.6% is needed to account for the loss of control attributable to the subject interest position, as if it were entirely noncontrolling.

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[13] See Dennis A. Webb, *Valuing Fractional Interests in Real Estate 2.0* (Los Angeles: Milonguero Press), 154–155 concerning the WACC adjustment process.

LLCs and LPs

Management adjustment

REIT management is generally considered competent, usually has the confidence of investors, and has a reliable succession plan. To the extent that the subject Company has more limited management capacity or faces challenges (based on the demands of their real estate assets), this yield premium may have to be increased to account for the greater risk.

A method of addressing partnership management risks is presented in a seminal book[14] that offers a five-level scale for assessing risk and applying adjustments. The levels are described as follows:

Level 5:   REIT management is generally considered to be the top category. REIT managers are likely to fee out responsibility for day-to-day operations (or have extensive in-house operations), but still make sure the properties are competently managed. Principally asset managers, they make institutional-level portfolio decisions, exiting and entering markets in response to external conditions. Key man risk for institutional management is generally minimal. RELP managers can also be at this level, but their competence may depend on property type.

   This level would also include managers who consistently outperform the market for the general property type. It is not all that unusual for an experienced local real estate investor to outperform REITs in his or her market, as local market expertise is sometimes absolutely critical to success. However, there is also no shortage of examples of poor local real estate decisions made by distant asset managers, professional though they may be. Some property types, like mobile home parks for example, demand local expertise market-wide. A private partnership with this sort of manager could be assigned Level 5, but there would also have to be some sort of key man assurance.

Level 4:   Applies generally to public limited partnerships and to some REITs holding difficult property types. This level requires management with a proven track record, good talent that does not rely on a single key man—with a strong operating agreement and buy-in from the partners. If the portfolio has properties in different markets, then having local property management may be important.

Level 3:   This level means average management, with some possibility of risk related to unusual and unforeseen conditions as well as key man reliance. This level might be appropriate for a good manager (Level 4) who is facing future conditions for which he or she might not be as well prepared as most investors in that market. It might also be appropriate for good management that is facing concentrated market risk that is not captured in the valuation of the underlying properties (see the diversification discussion later in this chapter).

Level 2:   Indicates marginal or unproven ability, known conditions for which the partnership is unprepared, or other unsettling conditions, which could be

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[14] See Webb, supra: 131–134, 161–163.

property- or consensus-related. However, poor performance has not yet been demonstrated. Such Poor performance could include possible conflicts with others—for example, with a manager constrained by existing limited partners, hampering his or her ability to respond to changed conditions. Succession offers little hope of any improvement.

Level 1: Applies when management has demonstrated poor decisions or does not inspire confidence that any issues (expected or otherwise) would not adversely affect the property in the future. Succession is generally uncontrolled. It is uncertain whether a buyer would enter such a partnership at any price, but the valuer is stuck with hypothesizing what the market would do anyway.

Management conclusions

Level 5 requires no adjustment to our earlier-concluded REIT-based yield premium. Each level below that adds 1.0% to the yield. In this case, the subject risk is slightly greater than it would be for a typical REIT, at Level 4. [Provide more reasoning in the body of the report]

The concluded control yield premium for the Company, to which we have assigned Management Level 4.0, is:

Control yield premium = REIT premium + management adjustment

Control yield premium = 2.6% + 1.0% = 3.6%

This conclusion is carried back to the body of the report, § IV-A, Control Impairments.

Subject degree of control

This heading is provided as a supplement, showing detail in support of our analysis of the degree of influence that the subject interest has in the Company in § IV-A, Subject Influence.

The degrees of control described below rely on an evaluation of the rights attributable to the subject interest and their potential exercise, given all the facts and circumstances of partnership organization as well as management demands and risks created by the property itself. Evaluation criteria can be divided under three separate headings, listed below.

- *Swing Vote* refers to the ability of the subject interest holder to combine its vote with at least one other to influence any decisions. The vote's value will certainly depend on the range of decisions required based on the property type and its circumstances. The likelihood of successfully combining with another depends on (1) the distribution of ownership, i.e., the number of others from which the holder may choose, and (2) whether their personal interests are likely to align with the subject in any particular circumstance.

- *Forcing Vote* means that the interest holder has the ability to make its own operating and other decisions. In this circumstance it would be wise to consider whether such decisions might be opposed by any of the other interest holders.

- *Blocking Vote* can be powerful, but also risky, since it very likely places the subject interest holder at odds with the manager/general partner and maybe other partners as well. The potential unintended consequences of attempted exercise should be carefully considered.

Subject interest control classifications

Just as for the complex issue of adjusting for management ability, valuers also need a structure or tools they can use to adjust the subject interest holder's ability to reduce its control-related risk. This list shows control attributes broken down into five degrees, or levels, of influence. It is not an entirely clean list because there are many potential interactions between identifiable risks, unknown risks, and how attempting to influence management would actually work out.

Degree	Swing Vote	Forcing Vote	Blocking Vote	Premium Reduction
5th Degree	N/A	Yes	Yes	60–90%
4th Degree	N/A	None	Yes	40–60%
3rd Degree	Yes, strong	None	None	20–40%
2nd Degree	Yes, but limited to daily operations	None	None	5–20%
1st Degree	Unlikely	None	None	0–5%
None	None	None	None	add 0–10%

5th Degree: This very high degree of control occurs when the interest holder can force sale of partnership assets with few obstructions. (If there were no obstructions whatsoever, then there would theoretically be no discount for lack of control, since the interest would have the same property rights as an outright holder of the real estate. Such a circumstance within a partnership is rare, though, as there can be many entanglements that would at least slow down such unilateral action. It is also likely that at least some discount for lack of marketability would apply because of potential delays.) The valuer should also consider whether a forced sale would necessarily result in distribution of cash proceeds. It would not be unusual for a forced termination of the partnership to result in distribution of its assets in kind, meaning conversion of partners' positions to common tenancy interests, which would then carry their own, less organized restrictions on control and marketability.

4th Degree: The interest holder has a very strong position because its vote is needed for major decisions, such as obtaining mortgage financing, making capital expenditures and selling assets. It cannot take such actions on its own (or it would have 5th Degree control), but it can block actions that the manager and other partners might wish to take. The importance of such blocking ability will vary as a function of known current or future circumstances where major decisions will be needed. Such an ability can also be a double-edged sword, as it almost guarantees internal conflict and could generate bad feelings at best or

lawsuits at worst. Considering all the facts and circumstances is especially important for the 4th Degree.

3rd Degree: The subject interest holder moves down to 3rd Degree control if it cannot rely on unilateral action but must rely on combining its vote with one other to take any meaningful steps toward 4th or 5th Degree control. It has a swing vote, which is more powerful if distribution of ownership is such that a swing combination can be made with just one of a number of other partners. A requirement to combine multiple votes is too speculative to place the interest at this Degree unless there were supporting facts.

2nd Degree: The interest holder can combine with one other to influence day-to-day operations (either directly or through the ability to replace the manager). It has a swing vote, as it does in the 3rd Degree, albeit a weaker one since it is limited to daily operations and cannot affect major decisions. This can occur, for example, when major decisions require a supermajority vote. Such control reduces risk in proportion to the management demands of the real estate.

1st Degree: This is the degree of control attributable to small interests in RELPs and REITs that have substantially zero control. It would be similar to the subject if there were no unanimous voting provisions. (With unanimous voting, the interest holder might automatically have the ability to block, say, termination of the partnership, sale of all its assets, or admission of a new partner.) The ensuing blocking right would then place the interest at a higher degree, depending on the likelihood that such a blocking right would become important.

None/Zero: Zero is not an actual degree of control as much as it is the complete absence of any possibility for control. This "degree" would normally apply to an assignee or economic interest, where the interest holder's sole right is to receive distributions. Such lack of any influence would normally be the same as the RELP or REIT investor's 1st Degree level; that is, unless not having partner status would deprive the interest holder of a right to, say, participate in a reorganized entity or something similar. This uncommon situation could slightly increase risk, dropping into the None/Zero category and adding to the discount for lack of control.

This list, like the management ranking system, can be expressed as a scale, as shown in the table. But instead of adding to a base yield premium as we did for the management adjustment, this table shows reductions from the concluded control premium.

For example, if the earlier-concluded control premium is 5.0%, then a third-degree level of control might reduce that premium by, say, 30% (the third-degree range is 20 – 40%). The premium would then be adjusted by 5.0% × 30.0% = 1.5%, and the adjusted premium would be 5.0% – 1.5% = 3.5%. Analysis is based on subject control characteristics described in § II-B, Company Operating Agreement, and adjustments are concluded in § IV-A, Subject Influence. [This is just an example; be sure to include your analysis in the body of the report.]

Common Tenancy

Management adjustment

REIT management is generally considered competent, usually has the confidence of investors, and has a reliable succession plan. To the extent that the subject cotenancy has more limited management capacity or faces challenges (based on the demands of the real estate asset), this yield premium may have to be increased to account for the greater risk.

A method of addressing common tenancy management risks is presented in a seminal book[15] that offers a five-level scale for assessing risk and applying adjustments. The levels are described as follows:

Level 5:   REIT management is generally considered to be the top category, with extensive in-house operations and succession that is generally assured. This level can also include managers who consistently outperform the market for the general property type. A common tenancy ownership group with this sort of management ability might be assigned Level 5, but there would also have to be key man assurance and a credible succession plan. If manager decisions require consensus, and if transfer is uncontrolled (both features of cotenancy without an operating agreement), then this level would be impossible to achieve.

Level 4:   This level requires management with a proven track record—good talent that does not rely on a single key man, normally with a strong operating agreement and buy-in by the partners. With no operating agreement, this level, like Level 5, would be nearly impossible for a cotenancy.

Level 3:   This level means average management and succession, with some possibility of risk related to unusual and unforeseen conditions as well as key man reliance. This level could also apply to good managers who are facing future conditions for which they might not be as well prepared as most investors in that market. This level should apply to most well-run common tenancy situations that do not have agreements to foster greater stability.

Level 2:   This level indicates marginal or unproven ability, known conditions for which the partnership is unprepared, or other unsettling conditions, which could be property- or consensus-related. Poor performance could include potential conflicts with others—and doesn't need to have been demonstrated in the past to still be a risk with an uncontrolled future transfer. It is not a big stretch to imagine cotenancy circumstances at this level that would make it impossible for the group to respond to changed conditions.

Level 1:   This level applies when decision-making conflict or failure has actually been demonstrated, or in cases where there are established concerns that do not inspire confidence. There might also be future issues that could adversely affect the property in the future. Succession is generally uncontrolled. As for the similar partnership situation, it is uncertain whether a buyer would enter such an ownership arrangement at any price, but the valuer is stuck with hypothesizing what the market would do anyway.

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[15] See Webb, supra: 254–256.

Conclusions

Level 5 requires no adjustment to our earlier-concluded REIT-based yield premium. Each level below that adds 1.0% to the yield. In this case, the subject risk is slightly greater than it would be for a typical REIT, at Level 4. [Provide more reasoning in the body of the report]

The concluded control yield premium for the Cotenancy, to which we have assigned Management Level 3.0, is:

Control yield premium = REIT premium + management adjustment

Control yield premium = 2.6% + 2.0% = 4.6%

This conclusion is carried back to the body of the report, § IV-A, Control Impairments.

Subject degree of control

This heading is provided as a supplement, showing detail in support of our analysis of the degree of influence that the subject interest has in the Cotenancy, in § IV-A, Subject Influence.

The degrees of control described here rely on an evaluation of the rights attributable to the subject cotenancy interest and their potential exercise, given the management demands and risks created by the property itself. Any of these criteria may be modified by agreement, creating a spectrum of organizational restrictions. The most complete and restrictive agreements may require evaluation as a general partnership or limited partnership rather than a cotenancy. Evaluation criteria can be generally divided under three separate headings. Any of these criteria may be modified by agreement, creating a spectrum of organizational restrictions. The most complete and restrictive agreements may require evaluation as a general partnership or limited partnership rather than a cotenancy.

- *Swing Vote* refers to the ability of the subject interest holder to combine its vote with at least one other to influence any decisions. Swing votes are unusual for cotenancies, since all cotenants have equal rights, and consensus is usually required for major decisions.

- *Forcing Vote* means that the interest holder has the ability to make its own operating and other decisions. In this circumstance it would be wise to consider whether such decisions might be opposed by any of the other interest holders. Forcing votes are unusual for cotenancies, as above, except that (assuming it is economically feasible), threatening to bring a partition action to force sale or division of the property is definitely forcing. It is not without risk.

- *Blocking Vote* can be powerful, but also risky, since it very likely places the subject interest holder at odds with the other cotenants. Blocking votes are unusual for cotenancies, as above.

Subject interest control classifications

Just as for the complex issue of adjusting for management ability, valuers also need a structure or tools they can use to adjust the subject interest holder's ability to reduce its control-related risk. This list shows control attributes broken down into five degrees of influence. It is not an

entirely clean list because there are many potential interactions between identifiable risks, unknown risks, and the question of how attempting to influence management would actually work out.

Degree	Swing Vote	Forcing Vote	Blocking Vote	Premium Reduction
5th Degree	Not realistically available to common tenants			60–90%
4th Degree	N/A	Financially feasible/no serious opposition	Yes	40–60%
3rd Degree	Realistic combinations	Financially feasible	Yes, but others are likely to block too	20–40%
2nd Degree	Yes, but limited to daily operations	No, or with strong opposition	Yes, but no foreseeable use	5–20%
1st Degree	Demonstrated decision-making conflicts or failures			0–5%
None	Almost impossible for common tenants			add 0–10%

5th Degree: This very high degree of control occurs when the interest holder can force sale of the real estate with few obstructions. This would only be possible if the other co-tenants had waived substantially all of their rights, in which case the organization would resemble a partnership and should be valued as such. Otherwise, this degree of control is not available to a cotenant.

4th Degree: At this level of control, the interest holder has a very strong position because its vote is needed for major decisions, such as obtaining mortgage financing (if this is indeed possible), making capital expenditures and selling assets. It cannot take such actions on its own (or it would have 5th Degree control), but it can block actions that the other cotenants might wish to take. The right to force partition might also confer some ability to control day-to-day operations, but such right would have to be financially feasible. This degree of control is typical for a cotenant, but its importance and degree ranking will vary as a function of known current or future circumstances where major decisions will be needed. As for a partnership, such ability can be a double-edged sword, as it almost guarantees internal conflict and could generate bad feelings at best or lawsuits at worst. Reflecting on all the facts and circumstances is especially important for the 4th Degree.

3rd Degree: The subject interest holder moves down to 3rd Degree control in a partnership if it cannot rely on unilateral action but must rely on combining its vote with at least one other: a swing vote. Such a swing vote normally has no meaning for cotenants, since each has an equal set of rights, but could apply if a voting structure were created by agreement. This degree of control could also apply

if the real estate were to require no decisions for many years and management was rated less risky as a result. The cotenant's blocking ability might only have meaning at the end of, say, a long remaining lease term, and thus would only be worth a 3rd Degree ranking instead of 4th Degree. The 3rd Degree might also be the result of a strong exercise of others' blocking ability, resulting in their de facto domination of the cotenancy.

2nd Degree: The cotenant still has a blocking right, but it has little practical effect because major decisions are not expected for a long time, and/or any attempts to force de-sired actions are hollow because the interest is so small that bringing an action to force partition is simply not financially feasible. While a larger-value interest could almost demand an exit if it wished, the holder of a small interest may very well be stuck for the duration. This degree could also result from others' willingness and financial ability to strongly oppose the subject's wishes. Not an attractive situation to be sure, but it still must be considered.

1st Degree: This degree is for substantially no control, which is not the case for any cotenant (except with a restrictive agreement, of course).

None/Zero: Zero is not an actual degree of control as much as it is the complete absence of any possibility for control. This "degree" would normally apply to an assignee or economic interest, where the interest holder's sole right is to receive distributions. As for the first degree, the zero degree is not normally applicable for cotenancy interests.

This list, like the management ranking system, can be expressed as a scale, as shown in the table. But instead of adding to a base yield premium as we did for the management adjustment, this table shows reductions from the concluded control premium.

For example, if the earlier-concluded control premium is 5.0%, then a third-degree level of control might reduce that premium by, say, 30% (the third-degree range is 20–40%). The premium would then be adjusted by 5.0% × 30.0% = 1.5%, and the adjusted premium would be 5.0% – 1.5% = 3.5%. Analysis is based on subject control characteristics described in § II-B, Company Operating Agreement, and adjustments are concluded in § IV-A, Subject Influence. [This is just an example; be sure to include your analysis in the body of the report.]

The Black-Scholes Option Pricing Model [SAMPLE]

The difficulty of capturing short-term market behavior is a major shortcoming of the income approach. This limitation can be solved, however, by adapting the Black-Scholes Option Pricing Model,[1] which is a good method for modeling short-term impairments. The model is normally used by valuers for pricing executive stock options and the like, but only for securities of actively traded companies for which price volatility can be measured. A lot has been written about the model's ability to predict marketability discounts,[2] but calculating the volatility term has been a roadblock. A recent book presents a methodology for resolving that roadblock, describing a way to transition between short-term (option) and long-term (investment) markets.[3] The result is a model that enhances the income approach, enabling it to account for very short periods and making it a true, across-the-board solution.

A put option allows the holder to sell specified shares, at a specified price and at a particular time in the future. If one holds shares that are not directly marketable and purchases a put option, then the holder has effectively purchased the shares' marketability. The price should therefore represent the discount for lack of marketability.

The principal variables in the Black-Scholes model are the price volatility of the stock, the prevailing interest rate, and the time to maturity of the option. This addresses the chief concern of one who cannot readily liquidate any security interest in a reasonable time—the likelihood of adverse price movements during the holding period. It mirrors the circumstances of market participants, in both the restricted stock and IPO studies, who cannot liquidate their positions for two to four (or more) years, and it also follows the logic advanced earlier with respect to the influence of expected volatility on the market discount.

The Black-Scholes put formula is somewhat complex, but it basically shows a discount from market price (cost of the put divided by the price of the stock) that varies directly with time and the volatility of the underlying stock, and inversely with the prevailing (risk-free) interest rate. The formulae are presented at the end of this appendix.

Short- and Long-Term Markets

Market value analysis demands connections with markets, either directly, through observations of transactions of similar interests, or indirectly, through proxies. Fractional interest valuation mostly involves proxy markets. The widely varying nature of fractional interests requires that

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[1] Fischer Black and Myron Scholes, "The Pricing of Options and Corporate Liabilities," *Journal of Political Economics* (May 1973): 637–659. This is the original publication, which was subsequently modified by Robert C. Merton of the Harvard Business School. Scholes and Merton (Black died in 1995) received the Nobel Prize in Economics for the model in 1997.
[2] David B. H. Chaffe III, "Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations," *Business Valuation Review* (December 1993): 182–188.
[3] Dennis A. Webb, *Valuing Fractional Interests in Real Estate 2.0*, (Milonguero Press, Los Angeles 2021): Chapter 12.

we consider two different proxy markets, depending on the expected holding/restriction period for the subject interest.

The first, and somewhat obvious, proxy market is the investor market. It is relatively long-term because of the investment objectives of the underlying real estate markets. Our principal sources of risk-related rates are the real estate markets, whose participants might expect to hold properties for as few as 2–3 years or as many as 15, but typically expect to hold them for anywhere from 5 to 10 years. Real estate investors also typically expect some change to take place, either in market conditions or for the property itself, but these changes do not normally happen quickly. Yield rates we are now using to value fractional interests rest on rates applicable to the underlying property, and such rates embed these investor holding period objectives. To apply rates developed from one set of circumstances to a subject with a different set of circumstances is to skate on very thin ice in both business valuation and real estate appraisal. Thus, short periods are typically not included in real estate yield rates; they are not even described by investor market data. Very long periods have much the same problem.

Figure E-1 – Discounts for Two Markets
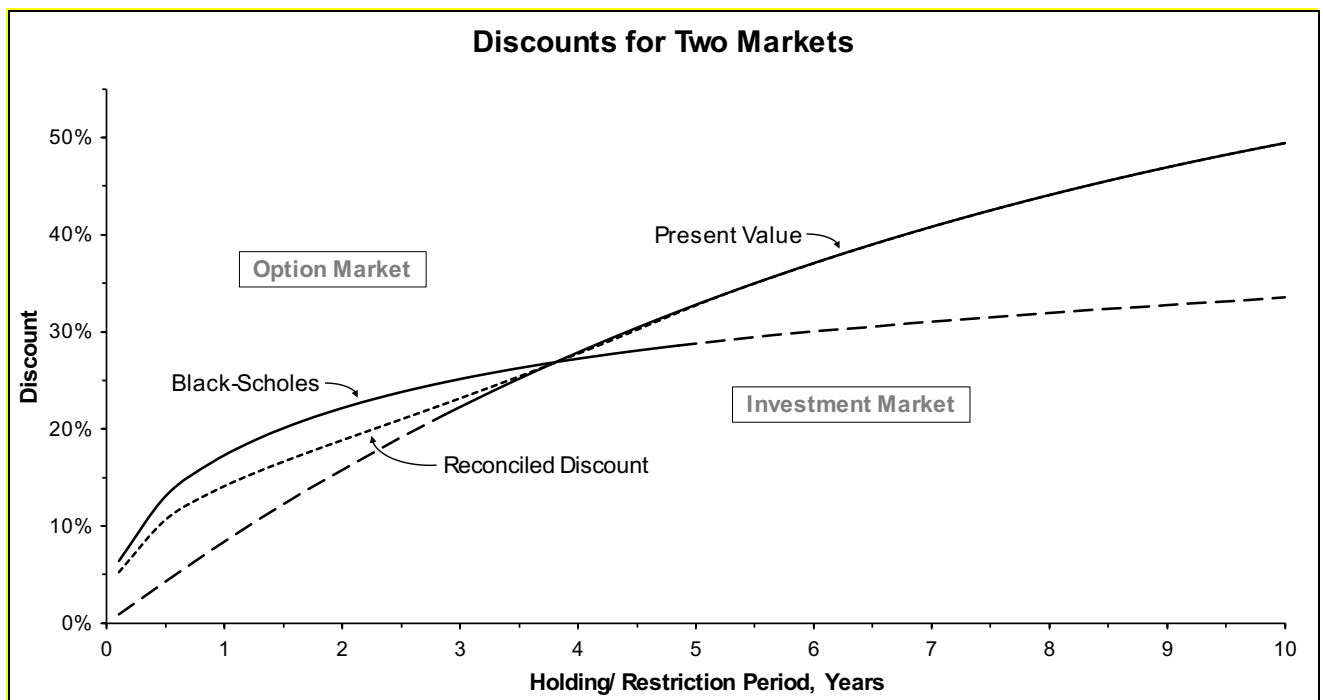
The second proxy market is the option market, which is, by its nature, short-term. The Black-Scholes Model is an empirical model intended to predict option pricing, and most options tend to have relatively near exercise dates, measured in months rather than years. The option market is a much better proxy for short-term fractional interest restriction periods than is the investment market.

If we are to have a complete picture of fractional interest pricing as a function of restriction period, we must somehow combine these two proxy markets. Their principal difference is the shape of their relationships to discount and time.

Figure E-1 shows the discount conclusions of both the present value and Black-Scholes models, based on typical cash flow and yield characteristics. The volatility term is estimated at 60% for the sake of illustration.

If we consider, for the moment, that a period shorter than four years is best analyzed as an option market, and a period greater than four years as an investment market, then the portion of the Black-Scholes line beyond four years is not really meaningful, nor is the portion of the present value line for fewer than four years. It is doubtful that the division between the two markets is so neat, however, and there is probably some investment market influence at three years, say, but much less at two years and hardly any at one year. We could say the same thing about the option market influence above four years; however, we have a good deal of confidence in the investment market concept at five years and above, making any Black-Scholes extension unnecessary. Thus, it is reasonable to ignore any option market influence above 4–5 years and apply this analysis only to the shorter periods.

The most interesting thing about Figure E-1 is the shape of the Black-Scholes curve for periods shorter than four years. The implication of the shape is that even a short delay increases risk dramatically, at a much faster rate than a present value model would show. This makes intuitive sense, and it is also consistent with short-period modeling from liquidating partnership studies.[4] An analysis of partnerships that have announced near-term liquidation shows discounts ranging from below 15% to more than 35% for expected holding periods ranging from 2–3 months to 18 months.

These discounts do need to be adjusted for distribution rates and the possibility of an exit during the short period, but they confirm that a very short period merits a much greater discount than a present value model would conclude.

The crossover point was selected at four years because the observed lower limit for real estate yield data is most often five years (as noted earlier) and is sometimes as low as two or three years. The latter is rare, making four years a good compromise. The selected point determines the equivalent volatility term needed to make the Black-Scholes line cross at that location. It is also the basis for our developing an empirical transfer function for converting yield to volatility. We are essentially "calibrating" the two markets to each other at that point, effectively extending the investment market discount curve to shorter periods by reshaping the present value curve according to the option market influence.

The figure's reconciliation line shows a small influence of the option market between years four and six, which begins to increase substantially below year four. The influences are almost equal around 2.5 years, but then the investor market's influence drops to a very small amount below year one. Discounts for every pair of valuation models will look something like this, the only requirement being that they cross at year four. The location of that point defines the volatility term.

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[4] Bruce A. Johnson, Spencer J. Jefferies and James R. Park. *Comprehensive Guide for the Valuation of Family Limited Partnerships*, 5th ed. (Dallas: Partnership Profiles, Inc., 2017): pages 41–43 and 151–161.

The Volatility Term

The primary obstruction to Black-Scholes modeling is its volatility term. Valuers can measure historic volatilities for companies whose shares are publicly traded and are able to measure implied volatilities for issues that have traded options. Valuers are also estimating volatilities for private companies, along the lines suggested by Chaffe: "The volatility of privately held shares can only be estimated by comparing the private company's financial and operating data with those of comparative public companies for which volatility can be determined. We note that volatility is low for large capitalization, actively traded issues and trends sharply upward for shares of smaller companies or highly speculative companies. This trend implies that volatility of shares of a small, privately held company might be at least 60%."[5] While this sort of estimating is clearly not very accurate, one could liken it to the yield rate adjustments routinely made for specific company risk—substantial adjustments based largely on the judgment of the valuer.

Figure E-1 suggests that we could develop an expression for volatility as a function of yield rate that would cause the Black-Scholes model to produce the same discount as the present value model for a holding/restriction period of four years, which is the effective intersection of option and investment markets. This effectively calibrates the Black-Scholes model, which describes option market behavior, to the present value model, which describes investor market behavior. This calibrating function was tested a number of ways for reasonableness, as described in our main reference text.[6]

The calibration process involves a present value (income) model and a Black-Scholes option model, both of which have inputs identical to those generated in the body of this report. The one exception is that the holding/restriction period is changed to four years. Calculations are, accordingly, almost identical to those shown at the end of this appendix (for Black-Scholes) and in the body of the valuation report for the income model and are thus not shown here. The option model's volatility term is iterated until the model's concluded discount is identical to the income model's concluded discount. The final volatility term is then used in the option model (below) with the report's concluded holding/restriction period, and the resulting discount indication is carried back to the body of the report. There, it is reconciled with the present value model's discount indication to calculate the report's concluded discount.

Calculations

The Black-Scholes formulae have been adapted for put option valuation by several authors, appearing slightly different in each adaptation depending on variable designations and minor algebraic manipulations. For example, Stephen Figlewski makes one of the clearest presentations and includes dividend payments, while Robert Jarrow and Andrew Rudd use a good, simple polynomial approximation for the standard normal distribution.[7]

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[5] Chaffe, *supra*, 184.

[6] Webb, *supra*, 323–327.

[7] Robert A. Jarrow and Andrew Rudd, *Option Pricing* (Homewood, IL: Richard Irwin, Inc., 1983): page 134.

The input variables required by the Black-Scholes formula are straightforward. The time to expiration of the option is the subject interest's restriction period. The volatility term is predetermined as the value that will make the present value and Black-Scholes discounts identical at four years. The dividend (distribution/cash flow), growth, and yield rates needed for incorporating dividends are basic variables that are concluded in the body of the report. The formulae presented on the next page are used to calculate the report's conclusions for this method.

Conclusions

The Black-Scholes Option Pricing Model was designed to price securities options, but the market behavior described by the model also fits observations of restricted marketability problems generally. It correlates reasonably well with restricted stock studies and provides us with a quantitative method for predicting the behavior of short-term markets.

Data supporting long-term market analysis generally involve investment periods from four to 10 or 15 years, with shorter periods being fairly rare. Restricted stock and securities options generally involve periods shorter than four to six years, with some as short as a few months. The most interesting thing about short-term markets is that they demonstrate a quite steep increase in risk for even very short periods, with the risk/time curve flattening toward four years. The relationship of risk to holding period that we are seeking clearly differs depending on the restriction period selected for a particular case.

The model's principal challenge is the volatility term, but the challenge is satisfied nicely if we postulate that the option and investment markets intersect at four years. This synthetic version is not analytically exact—pretty much nothing in valuation ever is—but it is consistent with market observations, and it ends up being a very useful tool.

Black-Scholes Formulae

[Paste UVS output here]